Photon Counting LADAR

M. J. Halmos, R. A. Reeder, B. Boland
M. J. Klotz, J. P. Bulot

Raytheon Space & Airborne Systems
El Segundo, CA 90245
What Is Photon Sensitive Receiver

- Receiver is capable of detecting single photon events
- Requires to have a very low receiver dark or thermal noise
- Intensity of return is often measured by counting single photon events
- Single photon Poisson statistics plays key role in determining the receiver statistics ($P_d$ and $P_{fa}$)
- Usually multiple pulses are transmitted to build enough counts for reliable detection
- Requires the least amount of transmitter power
Two Technical Approaches: Linear and Geiger Mode Avalanche Photodiodes

- **Geiger Mode APD**
  - Based on over-biased APD, where a photon event cause a controlled avalanche of carriers creating a substantial current or voltage
  - Most mature, arrays of 64x256 have been built
  - Simple ROIC
  - Needs to be reset after each detection event
  - Operates at both 1 µm and 1.5 µm regions

- **Linear Mode APD**
  - Low noise requirements favor HgCdTe electron only carrier APD, bypassing the McIntyre model for APD’s and producing an excess noise factor, F, of unity
  - Requires APD gains of 100’s to 1,000’s
  - Requires preamp and thresholding circuits (complex ROIC)
  - Preamp must be ultra low noise
  - New technology, arrays of 8x8 have been built
Concept of Operation For Geiger and Linear Mode APDs

<table>
<thead>
<tr>
<th>Detector Array</th>
<th>ROIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geiger Mode APD</strong></td>
<td><strong>Global 2 GHz Clock</strong>&lt;br&gt;<strong>Global Bias</strong>&lt;br&gt;<strong>Global Reset Circuitry</strong></td>
</tr>
<tr>
<td>Single photon</td>
<td>~ 3V</td>
</tr>
<tr>
<td>Detector Array</td>
<td>Stop Pulse&lt;br&gt;(no preamp noise)</td>
</tr>
<tr>
<td></td>
<td>Counter Register</td>
</tr>
</tbody>
</table>

**Global Controls** require one circuit for the whole array. **Local Controls** require circuits for each detector cell.

| **Linear APD** | **Global 2 GHz Clock**<br>**Local Bias Adjust**<br>**Local Threshold setting**<br>**Local Threshold crossing Electr.** |
| Single Photon | ~ pV |
| Detector Array | Preamp<br>Preamp Noise<br>SNR Reduction |
| Threshold | Counter Registers | Readout |
From Photon Poisson Statistics

\[ P_{\tau} \left( n \right) = e^{-\alpha \tau} \frac{\left( \alpha \tau \right)^n}{n!} \]
\[ \alpha \tau = N_t \quad \text{where} \quad \alpha = \text{Flux} \quad \& \quad \tau = \text{Time Bin} \]

\[ P_{\tau} \left( 0 \right) = e^{-N_t} \]
Probability of no photons arriving

\[ P_{\tau} \left( > 0 \right) = 1 - e^{-N_t} \]
Probability of one or more photons arriving

Including Speckle Statistics

\[ P_{\tau} \left( \mathcal{M} \right) = \left( \frac{\mathcal{M}}{\mathcal{M} + N_t} \right)^{\mathcal{M}} \rightarrow e^{-N_t} \quad \mathcal{M} \text{ is the number of speckle lobes} \]
Model for Geiger & Ideal Linear Developed From Poisson Statistics

**Linear**

\[ P_d = 1 - \left( \frac{M}{M + N_t} \right)^m e^{-n_n} \]

Clutter and dark counts do not affect signal detection

Probability that signal or noise trigger detection

**Geiger**

\[ P_d = \left[ 1 - \left( \frac{M}{M + N_t} \right)^m e^{-n_n} \right] \cdot \left( \frac{M}{M + N_c} \right)^m e^{-\frac{n_n J}{2}} \]

Probability that signal or noise trigger detection

Probability that no clutter occurred before the signal

Probability that no dark count occurred before the signal
Typical Operation for GMAPD is When Multiple Shots Are Used at Fraction of a Photon per Shot.

Transmitter design is such that a fraction of a photon is expected at the receiver per shot per detector.

\[ P_{d1} = \eta_{qe} \eta_{ph} \sim 0.35 \times 0.5 = 0.175 \]

We use about 6 total photons per measurement with \( n \sim 12 \) shots, the probability of a single detection would be,

\[ P_d = 1 - (1 - P_{d1})^n \sim 90\% \]

Probability of detecting \( \geq m \) of \( n \) pulse

\[ P_d = 1 - \sum_{j=0}^{m-1} \binom{n}{j} P_{d1}^j (1 - P_{d1})^{n-j} \]
Values Used For Calculations

\[ N_n = R_{\text{dark}} \tau \]  
Expected dark counts in one interval

\[ N_{t0} = 0.2 \]  
Expected number of photons per shot for target in the open

\[ \rho_t = 0.1 \]  
Target reflectivity

\[ \rho_c = 0.1 \]  
Clutter or obscurant reflectivity

\[ \eta_{ob} = 0.8 \]  
Target obscuration, \((1-\eta_{ob})\) can be seen

\[ M = 100 \]  
Number of speckle lobes on the detector element

\[ N_t = \frac{\rho_c}{\rho_t} \eta_{ob} N_{t0} \]  
Number of expected photons from the target per pulse

\[ N_t = \kappa - \eta_{ob} N_{t0} \]  
Number of expected photons from the obscurant or clutter

We assume that the \( P_d \) and \( P_{fa} \) (False Counts) for Geiger and Linear are exactly the same
Linear Is Typically not Affected by Obscurations

Geiger detection is inhibited by obscurations, but effect is mitigated by spreading the energy over multiple pulses. Linear detection would not suffer this limitation due to fast reset.
$P_d$ as Function of Total Photo-Electrons & Number of Pulses

90% target obscuration
$P_d = 80\%, 90\%, 95\%, \text{ and } 99\%$

**Geiger**
- $P_d = 99\%$
- $P_d = 80\%$

**Linear**
- $P_d = 99\%$
- $P_d = 80\%$

Region of LMAPD Advantage
- Geiger 95\% & 99\%
- Linear 95\% & 99\%
Effect on Increasing Noise To $P_d$ & $P_{fa}$

- No obscuration
- 0.019 expected photons per pulse from target
- 300 pulses per measurement
- $m = 2,3$ coincidence threshold detection
- 1,500 Range bins per gate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Line</th>
<th>Color</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_d$</td>
<td>LMAPD</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$P_{fa}$</td>
<td>GMAPD</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
Increasing The Number of Pulses Increases the Number of False Alarms

The probability of a false detection, $P_{fa}$, grows as $n^\gamma$ when $P_{fa}$ is $<< 1$.
Linear Mode APD Receiver

- So far we assumed that both LMAPD and GMAPD had similar single pulse characteristics:
  - $P_d \sim 35\%$ for single photon detection probability
  - False or dark count rate of $\sim 3$ kHz

- GMAPD detection chain is from the photon detecting APD to the range counter is only the detector, no additional signal conditioning circuitry.

- LMAPD detection chain consists of the photon detecting APD, Transimpedance Amplifier (TIA), threshold crossing circuitry that triggers the range counter. The whole chain must be considered when characterizing the receiver.

- We will develop the maximum noise tolerance of the TIA to keep the receiver chain within the performance of the GMAPD
  - Analytical Model – assumes Poisson distributed photons but perfect APD gain, no McIntyre statistics
  - Numerical Model – Including Poisson distributed photons and both perfect and McIntyre distributed gains
Analytical Model Using Perfect APD Gain

\[ P_d = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\sqrt{\text{SNR}} - \sqrt{\text{TNR}}}{\sqrt{2}} \right) \right] \]

\[ P_{fa} = \frac{1}{2} \left[ 1 - \text{erf} \left( \sqrt{\frac{\text{TNR}}{2}} \right) \right] \]

\[
\text{False_Counts} = \frac{P_{fa}}{\tau}
\]

\[
\text{SNR} = \left( \frac{2h\nu}{\tau \text{NEP}_{\text{TIA}}} \right)^2
\]

\[
\text{NEP}_{\text{TIA}} = \frac{i_{\text{TIA}} \sqrt{\text{BW}_n}}{\gamma M}
\]

### Summary from Graph

<table>
<thead>
<tr>
<th>M</th>
<th>pA/\sqrt{Hz}</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.18</td>
</tr>
<tr>
<td>500</td>
<td>0.8</td>
</tr>
<tr>
<td>1,000</td>
<td>1.6</td>
</tr>
<tr>
<td>2,000</td>
<td>3.2</td>
</tr>
</tbody>
</table>

3 kHz Dark Counts
Numerical Model of LMAPD

To model LMAPD detection we used 2 types of gain characteristics:

- Perfect gain – No distribution => Excess Noise Factor $F = 1$
  - so far only HgCdTe in electron only APD gain has shown this performance
- Gain distribution according to the McIntyre Model
  - best Excess Noise Factor possible is $F = 2$, but most materials are higher depending on the effective hole-to-electron ionization rate ratio $k_e$

\[
F \equiv \frac{\langle M^2 \rangle}{\langle M \rangle^2}
\]

Excess Noise Factor

McIntyre Probability Density Functions
(electron only ionization)

since $\sigma_M^2 = \langle M^2 \rangle - \langle M \rangle^2$ when $F = 1$ have $\sigma_M^2 = 0$

“perfect” gain in the APD world is a gain of exactly $M$
Numerical Modeling for Perfect Gain (F = 1) & McIntyre Gain with $k_e = 0$ (F=2)

**Perfect Gain** (e-APD HgCdTe)

- **M**
  - 100: 0.21
  - 500: 1.2
  - 1,000: 2.2
  - 2,000: 4.2

- **Ave. of 1000 Pulses**
  - Single Photo Electron
  - TNR = $4.5^2 = 20$
  - Low Pass = 1 GHz
  - High Pass = 48.6 MHz
  - Sampling = 2 GHz
  - False Alarm Rate = 6 kHz
Expected TIA Noise Requirements for Traditional APD Material Under McIntyre Statistics

HgCdTe will follow McIntyre statistics when designed to operate in other than electron only APD

Excess Noise Factor

\[ F = k_e M + \left(2 - \frac{1}{M}\right) \sqrt{2} - k_e \]

Ave. of 1000 Pulses
Single Photo-Electron
TNR = 4.5² = 20
Low Pass = 1 GHz
High Pass = 48.6 MHz
Sampling = 2 GHz
False Alarm Rate = 6 kHz
Experimental Small Array LMAPD Achieves Dark Count of ~ 600 kHz with Gain = 100

- Photoelectron detection efficiency is the probability that a photoelectron (created by a photon) will be detected
- Transimpedance gain = 40 mV/photon (@ G = 100)
- No Anti-reflection Coating
- Dark counts at Gain ~ 100 and $P_d \sim 35\%$ is ~ 600 kHz

Increasing Gain

Data at (80 Kelvin)

(Courtesy of Raytheon Vision Systems)
Numerical Example of 3 of n = 40 Coincidence Filtering with 30kHz False Counts

Point Cloud of Test Scene

Ladar Detection

Point Cloud of Test Scene 30 kHz Dark Counts but No Cross-talk Triggers

Point Cloud after 3 Detection Coincidence Filtering

Least 3 of 40 Coincidence Filtering
Conclusions

- We compared Linear vs Geiger, when $P_d$ and $P_{fa}$ for single pulse are the same
  - Under camouflage or clutter, Geiger approaches the linear performance when the number of pulses is high
  - Linear can operate with few pulses even under clutter and camouflage

- LMAPD can measure multiple pulses in a single range-bin, at the cost of more circuitry in the detector element unit cell

- Statistically, LMAPD has advantages over GMAPD when allowable measuring time is short and the target is obscured by clutter

- Efficient Linear Mode Photon Counting occurs when the detector material not be bound by McIntire statistics, which so far only HgCdTe seems to accomplish this

- In order for the LMAPD to match the GMAPD receiver, the preamp noise figure must be very low

- The achieved dark counts for the GMAPD are in the neighborhood of 3 kHz. For the same $P_d$ and $P_{fa}$ (per pulse) the LMAPD is in the order of 600 kHz. An improvement of about 200 is needed.

- The ROIC Circuitry needed for the LMAPD mode receiver is considerably higher then GMAPD, which may limit the size achievable in future arrays.