Compound speckle statistics in coherent lidars

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1. Introduction

During the last few decades, coherent lidar has established itself as a unique instrument in atmospheric remote sensing and its application area has broadened considerably along with the technologies contributing to it. Coherent lidars are currently employed in a large variety of atmospheric applications in fields as diverse as laser vibrometry and target identification, meteorological observation and atmospheric wind determination, aerosol and atmospheric constituents' concentration measurements, and tracking and control of pollutants atmospheric fluxes.

Many coherent lidar applications impose stringent power constraints while requiring high levels of sensitivity and accuracy. Therefore, to optimize the lidar system parameters ensuring full utilization of limited resources, it is of paramount importance to have a clear understanding of all possible sources of external disturbances, i.e., target speckle and atmospheric refractive turbulence, affecting lidar measurements. It is then natural that a problem of continuing interest among researches studying coherent lidar concerns the definition of a model for the physical mechanisms involved in the lidar problem and the statistical form of the lidar signal fluctuations.

In lidar systems, signal fluctuations could result from physical mechanisms other than target speckle, mainly refractive turbulence, so these mechanisms must also be the focus of this analysis. In general, fluctuations induced by turbulence are not as intense as those due to speckle in optical remote sensing systems but, although their normalized variance is smaller, they still need to be considered to properly describe the performance of any practical coherent lidar. Much work has been published on coherent lidar theory. Early analytical work on the problem concentrated on the reduction of heterodyne system efficiency caused by beam distortion under limited conditions that provide a reliable basis for preliminary assessment of lidar performance.[1,5] These mostly heuristic analyses statistically quantified turbulence-induced fading through its mean and variance, although they alone are not adequate to fully characterize system performance. Later analyses used in heterodyne lidar[6] attempted to overcome these limitations and fully characterize the statistics of heterodyne optical systems by defining theoretical expressions based on the path-integrated technique,[7] one of the theoretical asymptotic methods for estimating the higher moments of the fields propagated through random media. This technique predicts the leading-order effects of refractive turbulence with minimal approximations, providing a framework valid for any typical path-integrated atmospheric refractive turbulence. However, theoretical calculation of beam propagation and the higher moments of the field is still difficult and, consequently, no simple analytical solutions are known outside those obtained for simplified beam configurations and unrealistic atmospheric characterization. More recently, full-wave simulation of beam propagation has been used to examine the uncertainty inherent to the process of lidar heterodyne optical power measurement because of the presence of refractive turbulence for both passive receivers[8] and those using modal compensation of wave-front phase distortion.[9] By using the two-beam model, the lidar return is expressed in terms of the overlap integral of the transmitter and virtual (back-propagated) local oscillator beams at the target, reducing the problem to one of computing irradiance along the two propagation paths. Although the simulation of beam propagation permits the full statistical examination of the signal degradation in a heterodyne receiver caused by refractive turbulence under general atmospheric conditions and at arbitrary transmitter and receiver configurations, the technique is computationally demanding.

In this work,[10] our formulation circumvents the need for a detailed description of the compound speckle and turbulence problem. Such a specification is difficult because of the inherent complexity of the propagation as well as the random distribution of the atmospheric scatters. Instead, our phenomenological model first replaces the turbulence-distorted wavefront by an equivalent speckle representation of randomly dispersed coherent contributions. Then, compound the resulting turbulence fading statistics with target speckle statistics. The generalized K distribution, proposed in this study to describe fading return signals in heterodyne lidar receivers, has been derived under the assumption that the average coherent signal is a random variable driven by atmospheric turbulence. Now, the mean of the target speckle signal is smeared by turbulence speckle fluctuations and we need to define multiply stochastic (compound) statistics to describe the return signals in a coherent lidar (see Fig. 1).

2. Fading in coherent return signals

For a heterodyne receiver, with average power constraint P and noise power spectral density $N_0/2$, $\gamma_0=P/N_0B$ is the SNR per unit bandwidth B. The SNR γ_0 for a quantum or shot-noise limited signal can be interpreted as the detected number of photons (photocounts) per pulse when $1/B$ is the pulse period. Coherently detected signals are modeled as narrowband RF signals with additive white Gaussian noise (AWGN). For a heterodyne lidar system, in the presence of target speckle and atmospheric turbulence, we must consider fading signals, which are signals also affected by multiplicative noise. In the fading AWGN channel, we let α^2 denote the atmospheric channel power fading and $(P/N_0B)\alpha^2= \gamma_0\alpha^2$ denote the instantaneous received SNR per pulse. For a shot-noise-limited coherent optical receiver, the SNR of the envelope detector can be taken as the number of signal photons detected on the receiver aperture γ_0 multiplied by a heterodyne power mixing efficiency α^2: In addition to the effective delivery of the signal to the detector, the performance of the optical link also depends on the receiver sensitivity measured in terms of received photons. For systems with perfect spatial mode matching, the mixing efficiency is equal to 1. When the spatial modes are not properly matched, the contribution to the current signal from different parts of the receiver aperture can interfere destructively and result in the reduced instantaneous heterodyne mixing and consequent fading. Note that, conditional on a realization of the atmospheric channel described by α^2, this is an AWGN channel with instantaneous received SNR $\gamma=\gamma_0\alpha^2$. This quantity is a function of the random channel power fading α^2, and is therefore random. The statistical properties of the atmospheric random channel fade α, with mean square value $\Omega=\overline{\alpha^2}$ and probability density function (PDF) $p_\alpha(\alpha)$, provide a statistical characterization of the SNR $\gamma=\gamma_0\alpha^2$. In this study we define a statistical model for the fading amplitude α (i.e., SNR γ) of the received signal scattered by the atmospheric target after propagation through the atmosphere.

The presence of atmospheric turbulence needs to be considered into the statistical description of coherent lidar return signals. Turning attention to fading α, we study how amplitude and phase turbulence-induced fluctuations of the optical field define the statistics of the fading intensity α^2. We note that the random magnitude α must be expressed as an integral over the aperture and, hence, is the sum of contributions from each point in the aperture. In order to proceed with the analysis, we could consider a statistical model in which these continuous integral is expressed as a finite sum over N statistically independent cells in the aperture. Under the assumption that the number of independent coherent regions N is large enough, we can consider that α asymptotically approach a Rayleigh distribution.

Just as in a speckle pattern, the Rayleigh distribution for the turbulence amplitude fading length α is a consequence of the central-limit theorem. However, under conditions of weak-turbulence in which the number of coherent terms is small, the fading may actually be the result of summing a small number of terms. In this case, the fading α is not likely to be Rayleigh. Rather than assuming that α is always Rayleigh distributed for all conditions of turbulence, it is more realistic to assume that α satisfied a generalized Rayleigh distribution that becomes Rayleigh only in the limit as the number of coherent terms N becomes large. Such a distribution is the Nakagami-m distribution,[11] in essence a central chi-square distribution described by:

$$p_\alpha(\alpha)=2(mN)^m\ \frac{\alpha^{2m-1}}{\Gamma(m)}\ \exp\left(-mN\alpha^2\right) \quad (1)$$

Here, Γ is the complete gamma function. The parameter m characterizes the amount of turbulent fading. When $m\rightarrow 1$, the number of contribution coherent areas N is very large and the m-distribution reduces to Rayleigh. Note that the Nakagami-m distribution closely approximates the Rice distribution[11]. Applying the Jacobian of the transformation $\alpha^2=\gamma/\gamma_0$, the corresponding SNR γ distribution can be described according to a gamma distribution given by

$$p_\gamma(\gamma)=\left(\frac{mN}{\gamma_0}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-\frac{mN}{\gamma_0}\gamma\right) \qquad (2)$$

When $m\rightarrow 1$, the gamma distribution reduces to exponential distribution. The Nakagami-m parameter m and fading parameter N are a measure of turbulence effects.

The second principal performance limitation encountered by lidar receivers is produced by target speckle processes that appear because the backscattered signal is composed of a multitude of independent phased additive complex components. When only target speckle is considered, the integral of the complex field of a speckle pattern on the input aperture is, by the Central Limit Theorem -the number of random phasors contributing from a rough target to the speckle pattern is large,- a circular complex Gaussian random process over space.[12] It follows that the channel fading amplitude α must obey Rayleigh statistics

$$p_\alpha(\alpha) = 2M\alpha \exp\left(-M\alpha^2\right) \tag{3}$$

where M is the speckle fading mean-square value and represents the average number of speckles influencing the coherent measurement. Hence, the instantaneous SNR per pulse γ is distributed according to an exponential distribution given by

$$p_\gamma(\gamma) = \frac{M}{\gamma_0}\exp\left(-\frac{M}{\gamma_0}\gamma\right) \tag{4}$$

In a more general case of speckle, where the return signal is not defined by a single speckle pattern, but instead by the sum of n independent speckle patterns generated when n identical laser shots are averaged, the exponential distribution becomes a gamma density function of order n:[12]

$$p_\gamma(\gamma) = \left(\frac{nM}{\gamma_0}\right)^n \frac{\gamma^{n-1}}{\Gamma(n)}\exp\left(-nM\frac{\gamma}{\gamma_0}\right) \tag{5}$$

3. *Detection statistics in heterodyne lidars*

Detection of rough targets in a turbulent atmosphere requires that both the fluctuations of the target and the fluctuations of turbulence be taken into account (see Fig. 1). For typical atmospheric situations, the time scale of the fluctuations due to turbulence is several orders of magnitude larger than that of speckle-induced fluctuations (milliseconds rather than microseconds). The long time constant of the return signal fluctuation due to turbulence means that these fluctuations are essentially correlated over the short correlation time associated with speckle. Consequently, the mean of the target speckle signal is smeared by turbulence speckle fluctuations. There must be a compounding of the statistics for the signal affected by speckle Rayleigh fading conditioned on knowledge of the mean value as described by the Nakagami-m distribution characterizing the signal affected by turbulence. Now, the speckle process is driven by the turbulence random process and the problem must be analyzed by the application of conditional speckle statistics.

We can regard the speckle distribution Eq. (5) to be a conditional density function, conditioned on knowledge of the SNR γ_0, which we represent here by the variable x,

$$p_{\gamma|x}(\gamma|x) = \left(\frac{nM}{x}\right)^N \frac{\gamma^{n-1}}{\Gamma(n)}\exp\left(-nM\frac{\gamma}{x}\right) \tag{6}$$

The unconditional probability density function of the SNR γ is found by averaging the above density function with respect to the statistics of the conditional SNR x

$$\begin{aligned} p_\gamma(\gamma) &= \int_0^\infty p_{\gamma|x}(\gamma|x)\,p_x(x)\,dx \\ &= (nM)^n \frac{\gamma^{n-1}}{\Gamma(n)} \\ &\quad\times \int_0^\infty \left(\frac{1}{x}\right)^n \exp\left(-nM\frac{\gamma}{x}\right) p_x(x)\,dx \end{aligned} \tag{7}$$

By assuming that the speckle is driven by turbulence, we indicate that $p_x(x)$ obeys the gamma distribution Eq. (2)

$$p_x(x) = \left(\frac{mN}{\gamma_0}\right)^m \frac{x^{m-1}}{\Gamma(m)}\exp\left(-\frac{mN}{\gamma_0}x\right) \tag{8}$$

and, after some simple algebra, the result of the integration in Eq. (7) can be reduced to

$$\begin{aligned} p_\gamma(\gamma) &= \frac{2}{\Gamma(n)\Gamma(m)}\left(\frac{nM\,mN}{\gamma_0}\gamma\right)^{\frac{n+m}{2}}\frac{1}{\gamma} \\ &\quad\times \mathrm{K}_{m-n}\left[2\left(\frac{nM\,mN}{\gamma_0}\gamma\right)^{\frac{1}{2}}\right] \end{aligned} \tag{9}$$

Here, K_β is a modified Bessel function of the second kind, order β. This result is a generalization of the well-known K-distribution. Equation (9) is the main outcome of this modeling effort. The model results in a three-parameter probability distribution for the coherent signal-to-noise ratio in the presence of atmospheric turbulence and affected by target speckle. These parameters are the average number of speckles influencing the coherent measurement M, the number of turbulent coherent areas in the receiver affecting the fading measurement N, and the amount of fading introduced by atmospheric turbulence in the lidar signal m. These three parameters, along with the number of averaged shots n and the turbulence-free photocount budget γ_0, completely characterize the statistics of return fading signals in coherent lidars.

To have a measure of speckle and turbulence effects, it has been necessary to establish closed expressions and develop procedures to estimate M, N and m.[10] First, in order to assess the impact of turbulence, both log-amplitude and phase fluctuations have been considered in N and m. Then, the number of spatial speckle modes M depends on the correlation function of speckle and the receiving aperture function. More detailed results and comments on our analysis will be presented at the meeting.

4. Conclusion

A statistical model for the return signal in a coherent lidar is derived from the fundamental principles of atmospheric scattering and turbulent propagation. The model results in a three-parameter probability distribution for the coherent signal-to-noise ratio in the presence of atmospheric turbulence and affected by target speckle. We consider the effects of amplitude and phase fluctuations, in addition to local oscillator shot noise, for both single aperture receivers and those employing spatial diversity. We obtain exact expressions for statistical moments for lidar fading, and evaluate the impact of various parameters, including the ratio of receiver aperture diameter to the wave-front coherence diameter, the speckle effective area, and the lidar receiver configuration.

5. Acknowledgements

This study was partially funded by the Spanish Department of Science and Innovation MICINN Grant No. TEC 2009-10025.

6. References

1. D. L. Fried, "Optical heterodyne detection of an atmospherically distorted signal wave front," Proc. IEEE **55**, 57-67 (1967).
2. H. T. Yura, "Signal-to-noise ratio of heterodyne lidar systems in the presence of atmospheric turbulence," Optica Acta **26**, 627-644 (1979).
3. J. H. Shapiro, B. A. Capron, and R. C. Harney, "Imaging and target detection with a heterodyne-reception optical radar," Appl. Opt. **20**, 3292-3312 (1981).
4. S. F. Clifford and S. Wandzura, "Monostatic heterodyne lidar performance: the effect of the turbulent atmosphere," Appl. Opt. **20**, 514-516 (1981). Correction, Appl. Opt. **20**, 1502 (1981).
5. B. J. Rye, "Refractive-turbulent contribution to incoherent backscatter heterodyne lidar returns," J. Opt. Soc. Am. **71**, 687-691 (1981).
6. R.G. Frehlich and M. J. Kavaya, "Coherent laser radar performance for general atmospheric refractive turbulence," Appl. Opt. **30**, 5325-5352 (1991).
7. J. L. Codona, D. B. Creamer, S. M. Flatté, R. G. Frehlich, and F. S. Henyey, "Solution for the fourth moment of waves propagating in random media," Radio Sci. **21**, 929-948 (1986).
8. A. Belmonte and B. J. Rye, "Heterodyne lidar returns in turbulent atmosphere: performance evaluation of simulated systems," Appl. Opt. **39**, 2401-2411 (2000).
9. A. Belmonte, "Influence of atmospheric phase compensation on optical heterodyne power measurements," Opt. Express **16**, 6756-6767 (2008).
10. A. Belmonte, "Statistical model for fading return signals in coherent lidars," Appl. Opt. **49**, 6737-6748 (2010).
11. M. Nakagami, "The m-distribution. A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, W. C. Hoffman, ed. (Pergamon Press, 1960).
12. J. W. Goodman, *Speckle Phenomena in Optics. Theory and Applications* (Ben Roberts & Company, 2007).

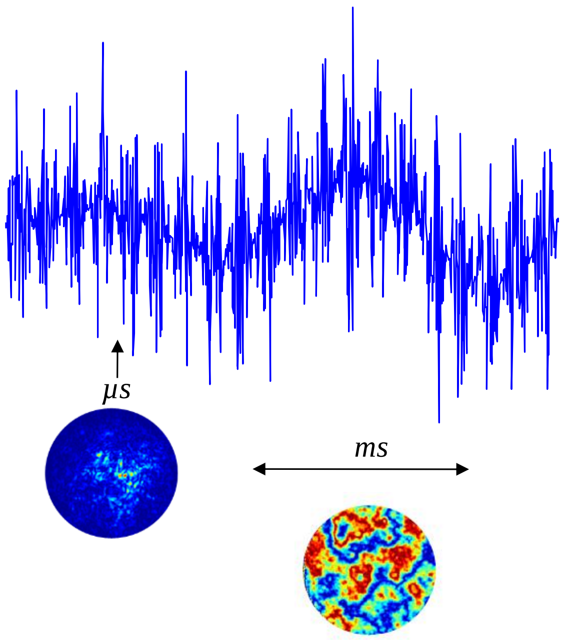

Fig. 1. For typical atmospheric situations, the time scale of the coherent lidar signal fluctuations due to turbulence is several orders of magnitude larger than that of speckle-induced fluctuations (milliseconds rather than microseconds). The long time constant of the return signal fluctuation due to turbulence means that these fluctuations are essentially correlated over the short correlation time associated with speckle. Now, the mean of the target speckle signal is smeared by turbulence speckle fluctuations and we need to define multiply stochastic (compound) statistics to describe the return signals in a coherent lidar.