Compound speckle statistics in coherent lidars

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INTRODUCTION

- Many coherent lidar applications impose stringent power constraints while requiring high levels of sensitivity and accuracy.

- It is of paramount importance to have a clear understanding of all disturbances affecting lidar measurements

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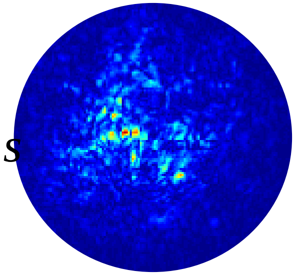
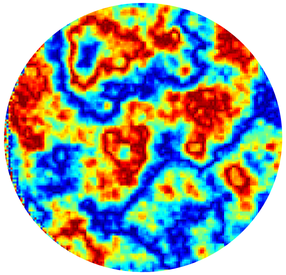
- Atmospheric turbulence coupled with target speckle can deteriorate coherent laser radar performance. For a heterodyne lidar system, we must consider fading signals, which are signals affected by multiplicative noise.

$\tau \sim 1\mu s$



Target Speckle



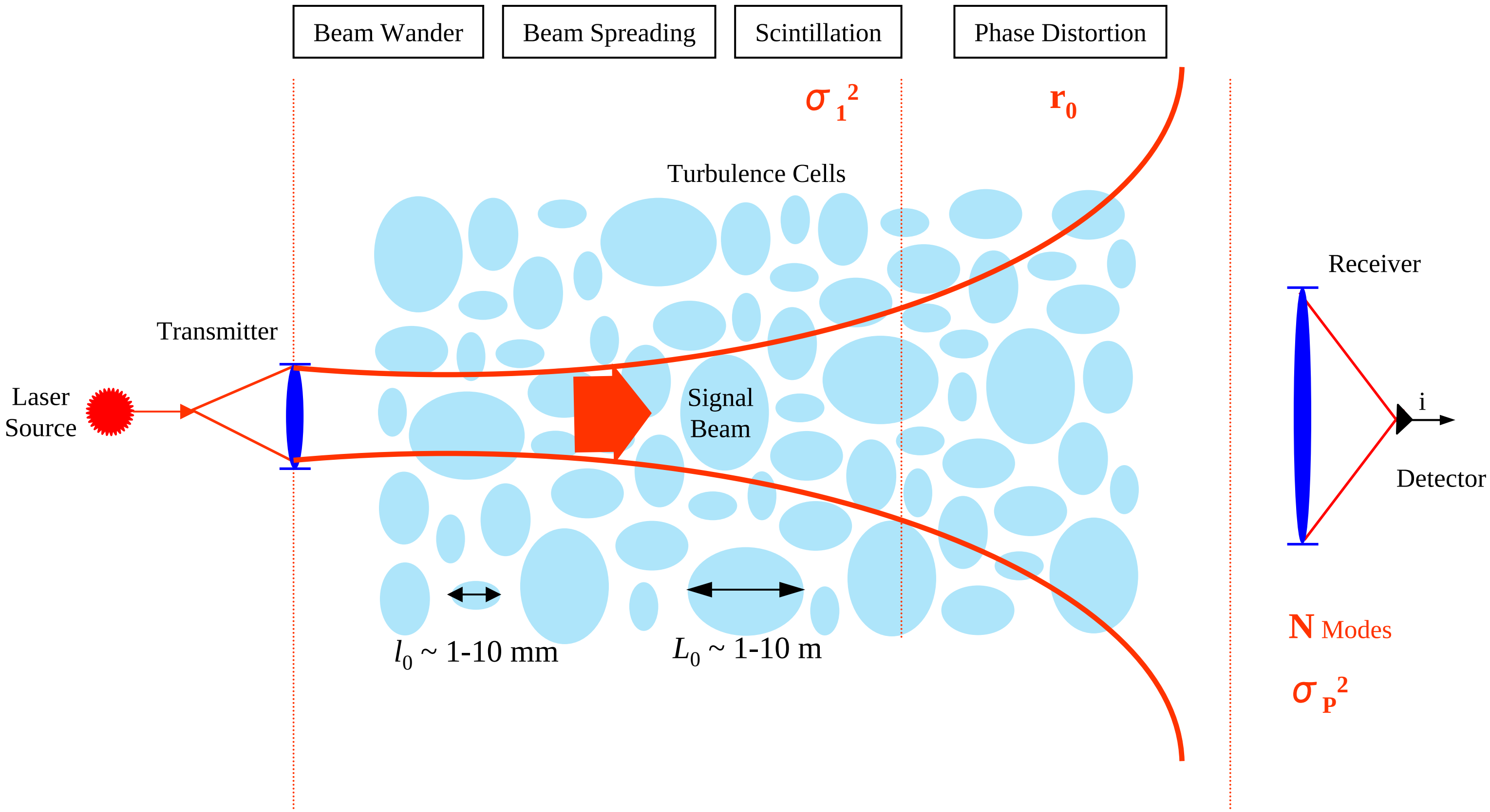
$\tau \sim 1ms$

Atmospheric Turbulence

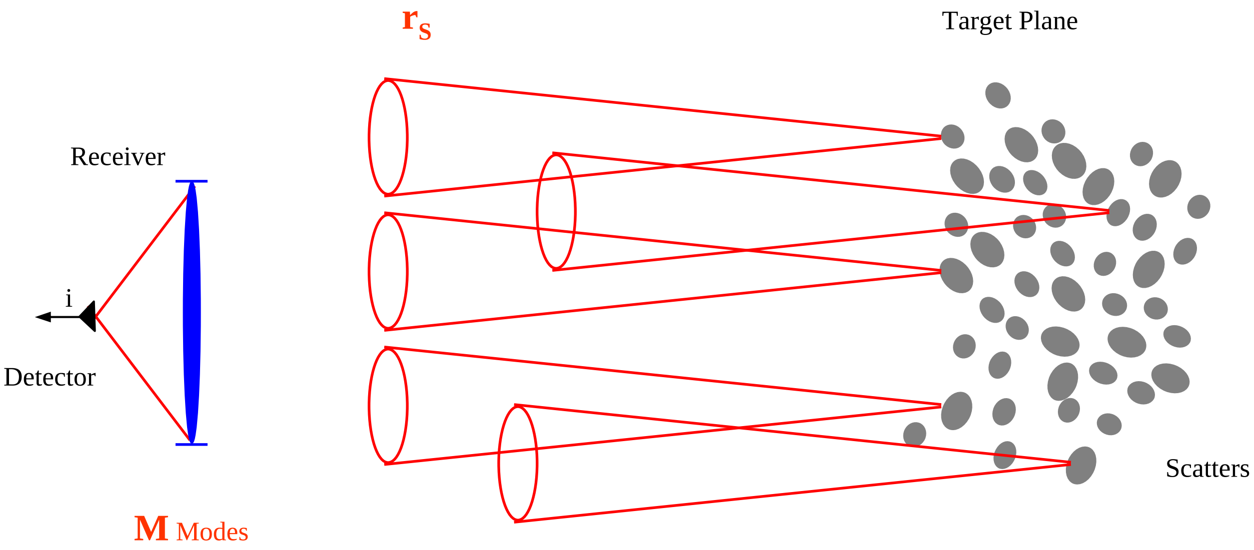
Outline

- Introduction: Atmospheric Turbulence and Target Speckle
- Compound Statistics for Fading Signals
- Applications: Characterization, Simulation, and Estimation
- Final Remarks

Atmospheric Turbulence Effects

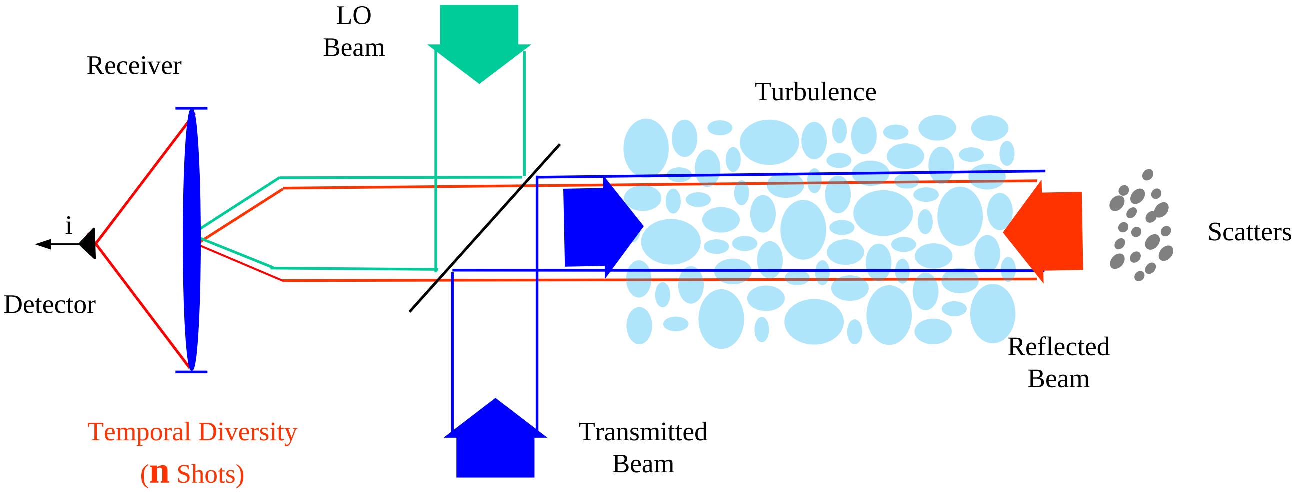


Target Speckle Effects



Lidar Coherent Systems

$$\gamma_0 = \frac{P}{N_0 B}$$



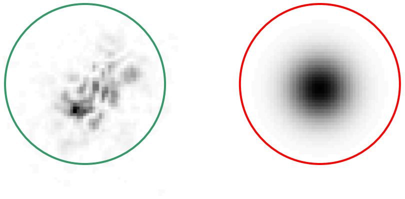
Temporal Diversity
(**n** Shots)

Signal-to-Noise Ratio and Fading

•The SNR γ_0 per pulse in the absence of turbulence and speckle is afected by fading. If α^2 denote the atmospheric channel power fading, the instantaneous received SNR per pulse is

$$\gamma = \gamma_0 \alpha^2$$

•We need to define a statistical model for the fading amplitude α (i.e., SNR γ) of the received signal scattered by the atmospheric target after propagation through the atmosphere:



$$\alpha = \frac{1}{cte} \int_{APERTURE} d\boldsymbol{r} \;\; E(\boldsymbol{r}) \;\; E_{LO}(\boldsymbol{r})$$

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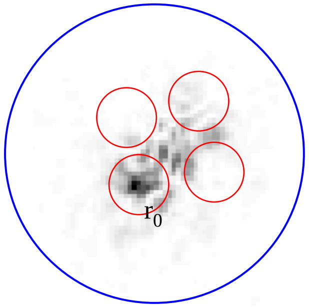
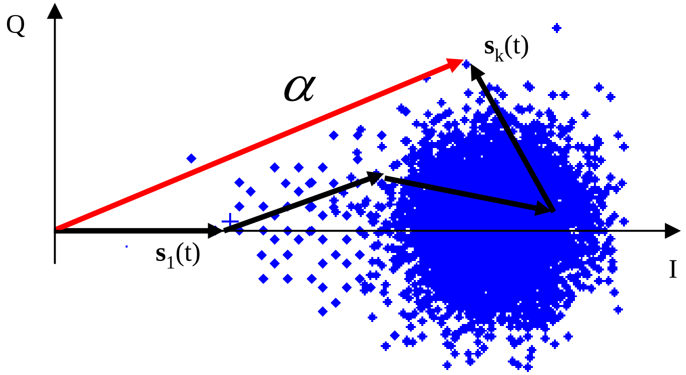
Turbulence Fading

- Under the assumption that the number of independent coherent regions N is large enough, the probability density function of α can be well approximate by a Rayleigh distribution.





Multiplicative Turbulence Noise

- Under conditions of weak-turbulence, the number of coherent terms is small, and it is more realistic to assume that α satisfied a generalized Rayleigh distribution that becomes Rayleigh only in the limit as the number of coherent terms N becomes large.

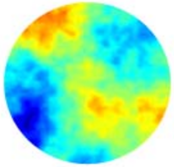
Turbulence Fading

•A generalized Rayleigh distribution for α is the Nakagami-m distribution and the corresponding SNR γ distribution, using the transformation $\gamma = \gamma_0 \alpha^2$, can be described according to a non-central chi-squared distribution distribution:

$$p_T(\gamma) = \left(\frac{mN}{\gamma_0}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{mN}{\gamma_0}\gamma\right)$$

$$N = \frac{1}{\langle \alpha^2 \rangle}$$

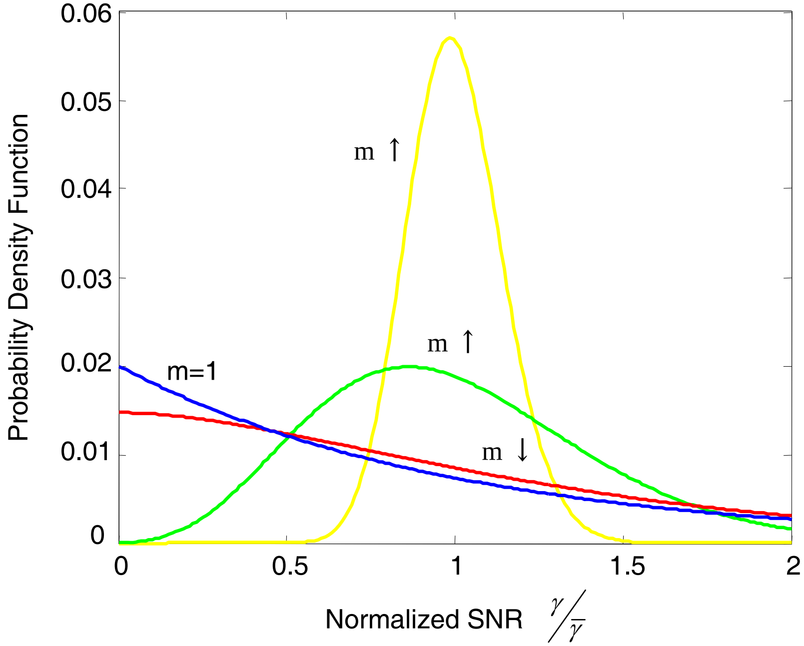
Number of turbulent coherent areas in the receiver affecting the fading measurement.

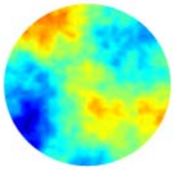


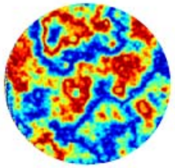
$$\frac{1}{m} = \frac{\sigma_\alpha^2}{\langle \alpha \rangle^2}$$

Amount of fading introduced by atmospheric turbulence in the lidar signal.

Turbulence SNR Statistics

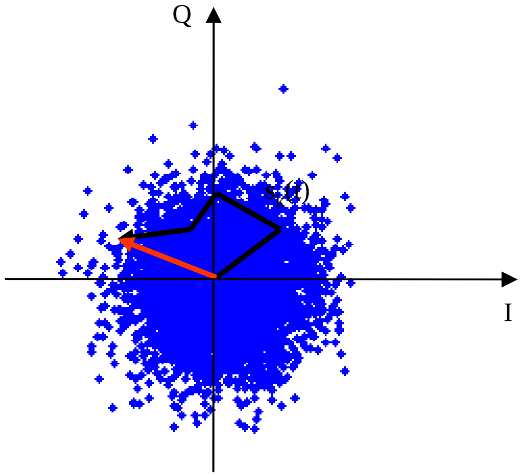



m ↑


m ↓

Speckle Fading

- The number of random phasors contributing from a rough target to the speckle pattern is large and, consequently, the speckle fading amplitude α must also obey Rayleigh statistics

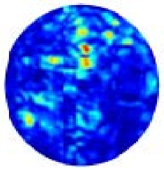


Multiplicative Speckle Noise

$$p_S(\alpha) = 2M\alpha \exp\left(-M\alpha^2\right)$$

$$M = \frac{1}{\left\langle \alpha^2 \right\rangle}$$

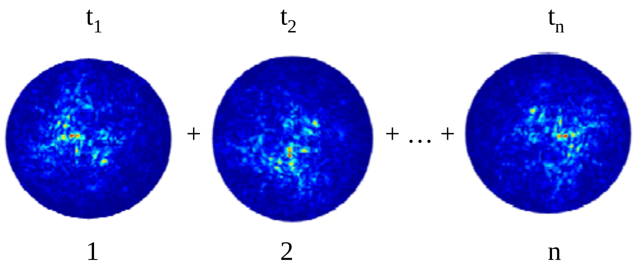
Average number of speckles influencing the coherent measurement



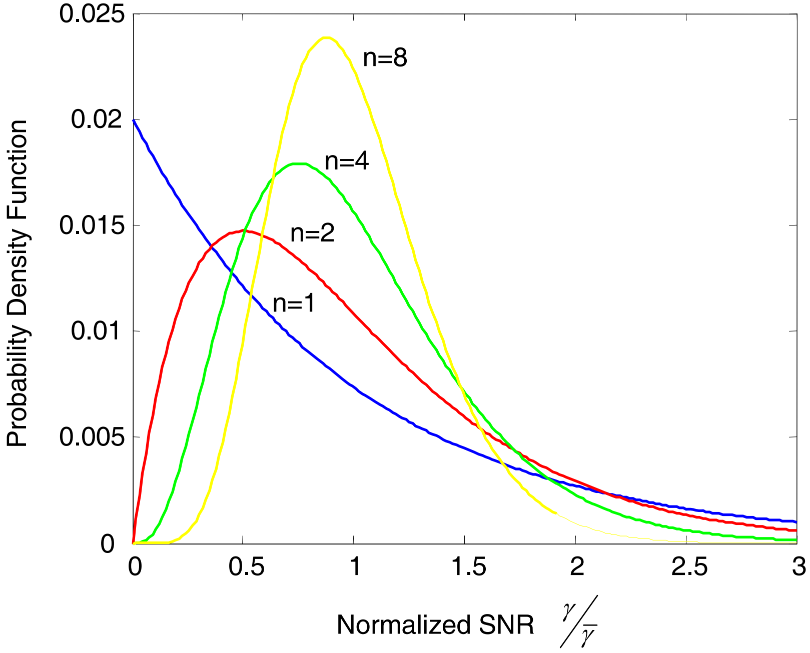
Speckle Fading

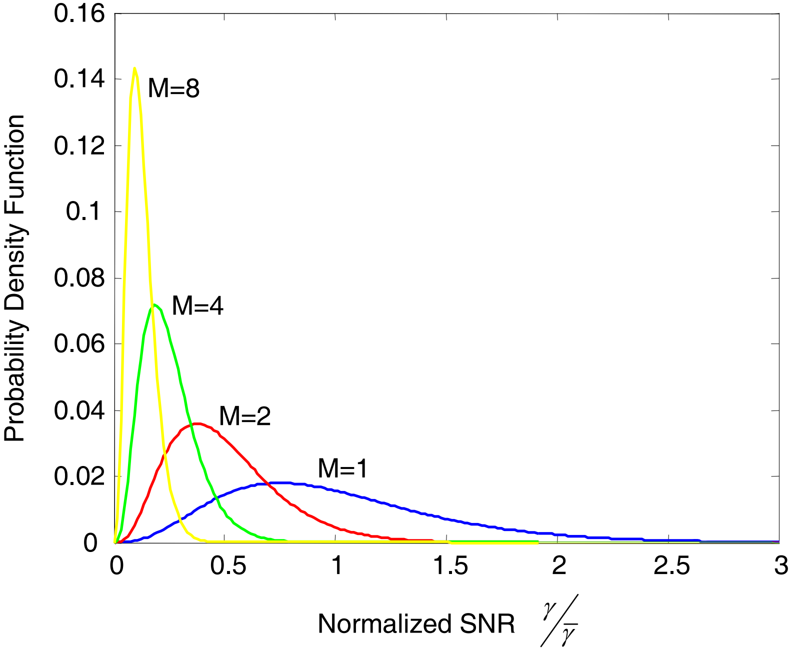
- The instantaneous SNR per pulse γ is distributed according to an exponential distribution or, if *n* accumulation shots are considered, γ is described by a Gamma distribution

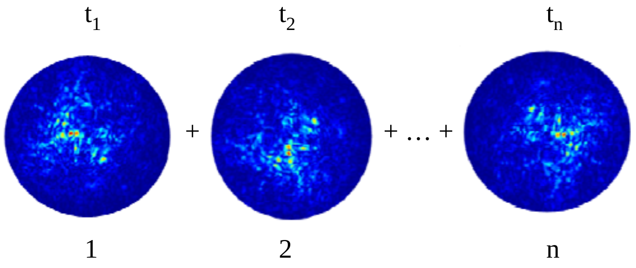
$$p_S(\gamma) = \left(\frac{nM}{\gamma_0} \right)^n \frac{\gamma^{n-1}}{\Gamma(n)} \exp\left(-nM \frac{\gamma}{\gamma_0} \right)$$

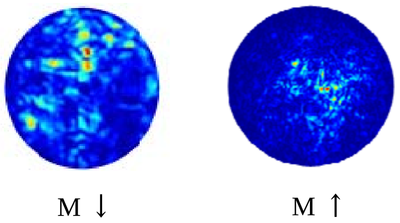


Speckle SNR Statistics



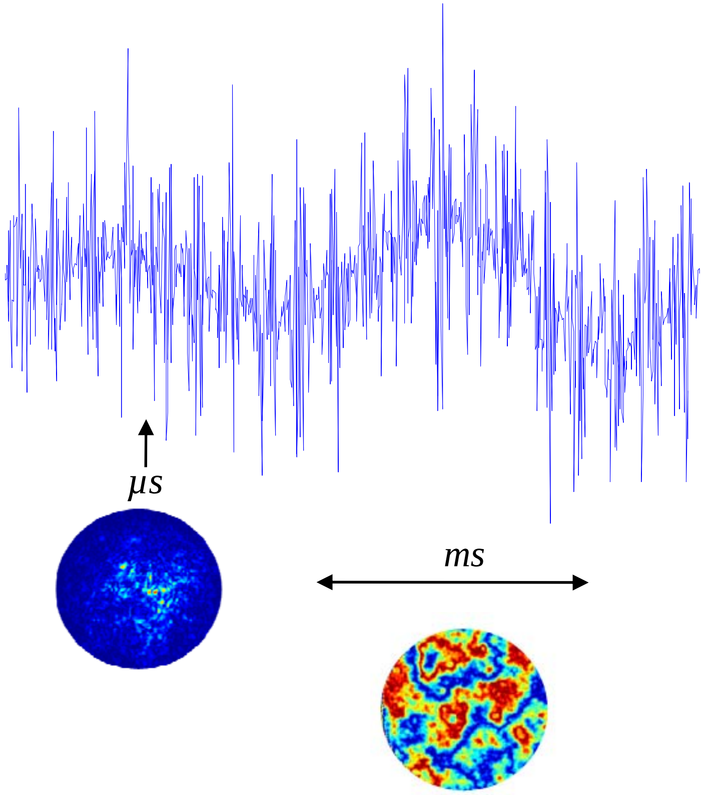






Speckle Driven By Turbulence

- The speckle process is driven by turbulence. The problem of compound speckle statistics can be analyzed by the application of conditional statistics.



$$p_\gamma(\gamma) = \int_0^\infty p_S(\gamma \mid v)\; p_T(v)\; d\gamma$$

Coherent Lidar Signal Statistics

- We suggest a continuous PDF from the family of K distributions, the gamma transform of a non-central chi-squared distribution, to model the smearing due to speckle of the SNR in a heterodyne lidar receiver affected by atmospheric turbulence.

$$\boxed{p_\gamma(\gamma)=\frac{2}{\Gamma(n)\Gamma(m)}\left(\frac{nM\,mN}{\gamma_0}\gamma\right)^{\frac{n+m}{2}}\frac{1}{\gamma}\,\mathrm{K}_{m-n}\left[2\left(\frac{nM\,mN}{\gamma_0}\gamma\right)^{\frac{1}{2}}\right]}$$

- Three parameters (m, N, and M), along with the number of averaged shots *n* and the turbulence-free photocount budget γ_0, completely characterize the statistics of return fading signals in the presence of atmospheric turbulence and affected by target speckle.

- Applied Optics, Vol. 49, Issue 35, pp. 6737-6748 (2010).

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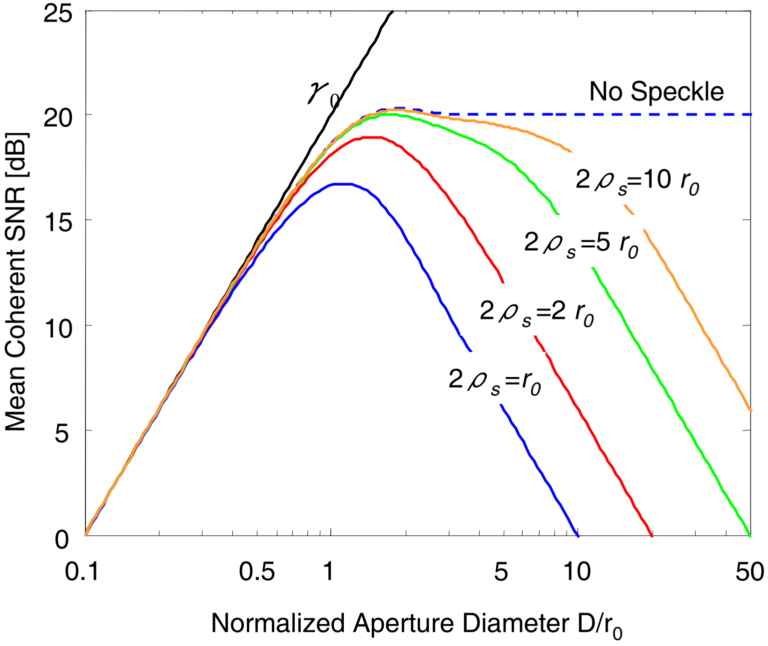
System Performance and Characterization

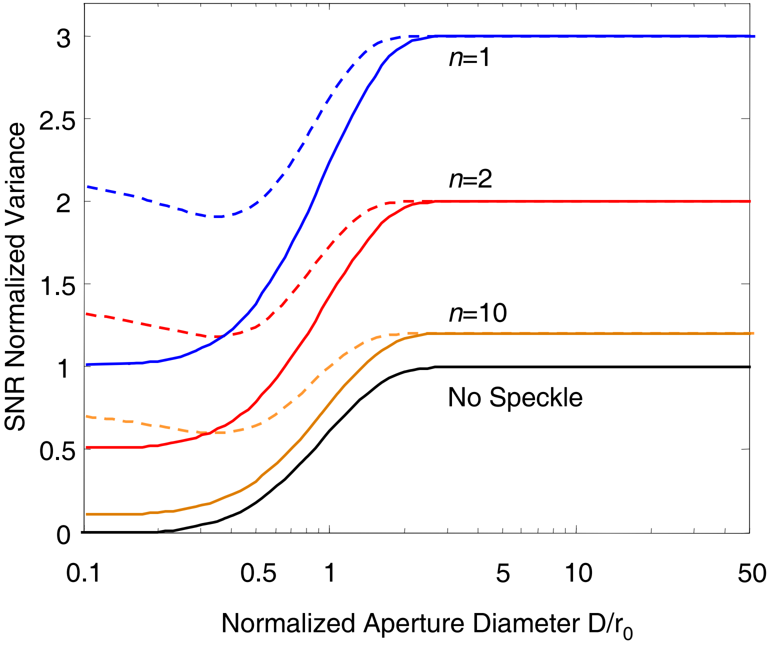
- Our model helps to explore the various possibilities that exists to design lidar coherent systems.

- Statistical measures of performance (e.g., mean SNR, amount of fading, outage probability) are required to characterize and estimate the effectiveness of the systems.

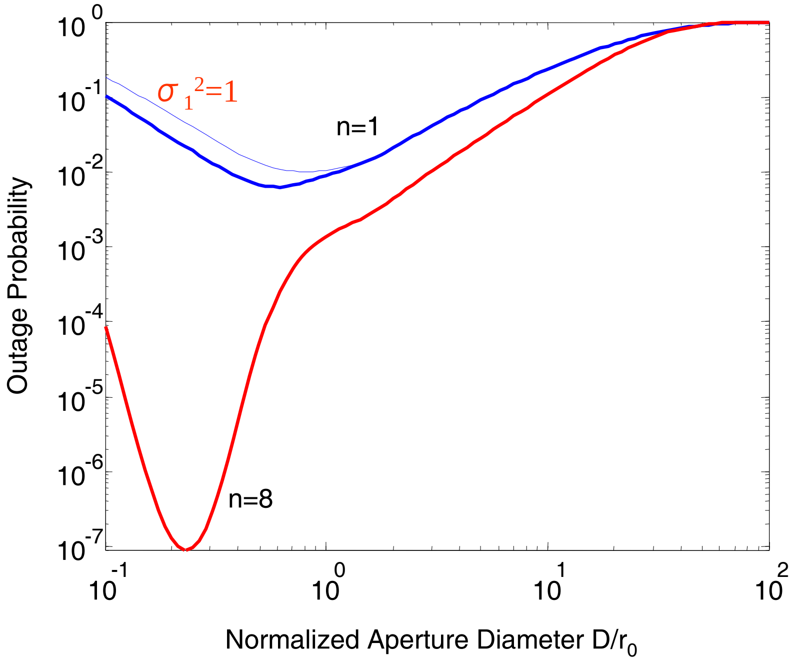
$$\left\langle \gamma \right\rangle = \int_0^\infty \gamma \; p_\gamma\left(\gamma\right) \, d\gamma \qquad \left\langle \gamma^2 \right\rangle = \int_0^\infty \gamma^2 \; p_\gamma\left(\gamma\right) \, d\gamma$$

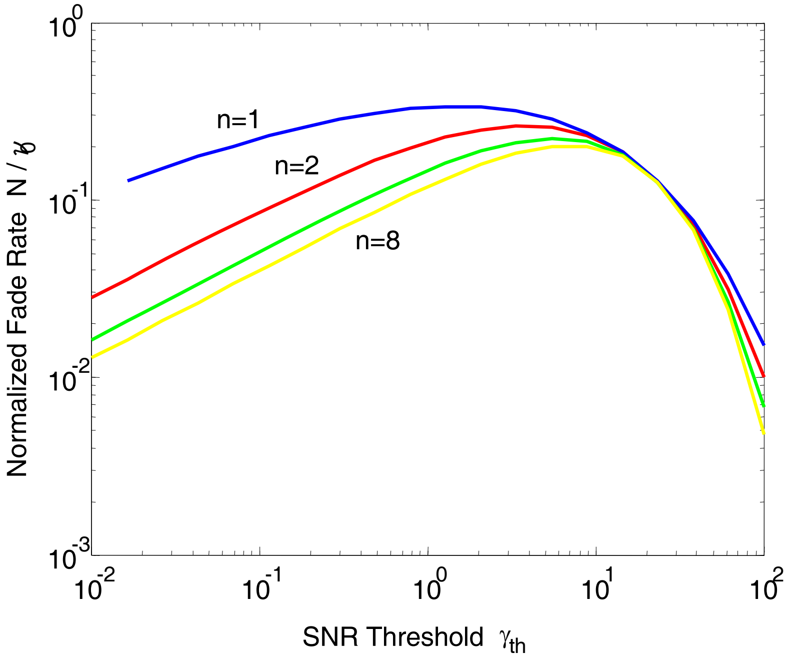
Mean SNR and Amount of Fading





Outage (Fade) Probability



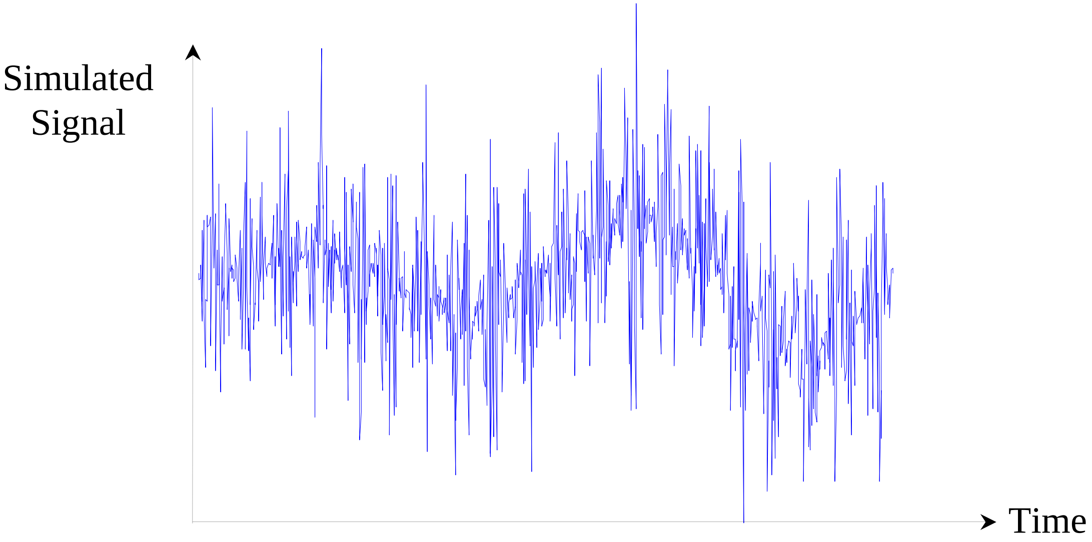


System Simulation

- The statistical model can be used to define a simulation model, and the corresponding algorithm, of the signal fading in coherent lidar systems.

- A simulated signal -random sequence with the characteristics of the compound K-distribution. Simulation results can be used to explore a broad set of system and atmospheric parameters.

- This simulation method is more effective than classical full-wave simulation of beam propagation.



Empirical Estimation

- Comparison of experimental data with the higher moments of the SNR distribution affords a sensitive measure of the model parameters m, N, and M.

- To explore the parameters, the empirical data can be fitted to the K distributions and their moments.

- We could estimate atmospheric parameters related to scatters and turbulence from them.

- A wide range of experimental data must be considered to check its physical basis. This experimental work constitute an entire field of study in itself.

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Final Remarks

•The speckle reflected from atmospheric scatters is a compounded effect of the turbulence-induced SNR fluctuations and the speckle fluctuations.

•A statistical model for the return signal in a coherent lidar is derived from the fundamental principles of atmospheric scattering and turbulent propagation.

•The model results in a three-parameter probability distribution for the coherent signal-to-noise ratio in the presence of atmospheric turbulence and affected by target speckle.

•Spatial and temporal diversity methods, along with phase correction techniques, can be applied to mitigate signal deterioration. Our statistical model helps to explore the various possibilities that exists to deploy compensation techniques.