Compensation of laser beam wave front aberrations based on atmospheric backscattering

V.A. Banakh, I.N. Smalikho
Zuev Institute of Atmospheric Optics SB RAS, 1 Ak. Zuev Square, Tomsk 634021
banakh@iao.ru , smalikho@iao.ru

1. Introduction

For high-altitude lidar sounding and free space optical communications, the influence of atmospheric turbulence is negligibly small and initial wave front distortions of the partially coherent laser beam play the decisive role in the reduction of the laser energy. In this connection, it becomes important to compensate initial aberrations of the beam wave front. In [1], the compensating the initial wave front aberrations (wave front collimating) of a cw laser beam based on the backscattered signal has been studied. The compensation method used in [1] assumes the splitting of the laser beam into the principal and probing beams. The backscattered signal from the probing beam detected by detector with a narrow angle of view field is used to control deformable mirror. The deformable mirror is deformed so that the divergence of probing beam become less and the backscattered signal increases. Since probing beam and principal one have the same wave front aberrations, which are corrected by the same deformable mirror, the principal beam improves its M^2 quality factor as well. This paper presents the results of study of the possibility of compensating aberrational wave front distortions by the same method in case of pulsed partially coherent laser beam. We also propose a new backscattering method for the compensation of wave front aberrations of a CW laser beam which does not require the splitting of the beam into the principal and probing ones.

2. Basic relations and simulation results

2.1. Pulsed laser

Figure 1 shows the schematic of the transceiving optical system with wave front aberration compensation based on the backscatter of probing beam. The laser source radiation is split into the principal and probing beams. With the use of the lens telescope, the probing beam is focused through the transmitting ring aperture at some distance. The radiation scattered in the probed volume (near the focus) is received by the circular aperture and comes to a photodetector at a certain field-of-view angle. The mean power of received backscattered radiation serves as a control signal for the deformable mirror, whose surface changes to decrease initial wave front aberrations and thus to maximize the received signal power.

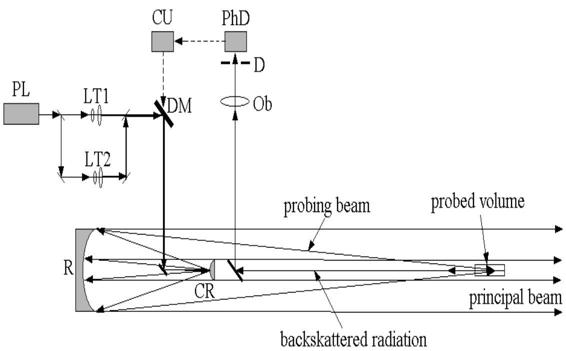

Fig. 1. Schematic diagram of the setup: pulsed laser PL, lens telescopes for the principal and probing beams LT1 and LT2, deformable mirror DM, contrreflector CR and reflector R of the transmitting telescope, objective Ob of the receiving telescope, diaphragm D, photodetector PhD, deformable mirror control unit CU.

For the mean power $<P_s(x)>$ of the radiation scattered from the probed volume at a distance $x > \Delta p$ ($\Delta p = c\tau_P/2$, c is the speed of light, τ_P is the probing pulse duration) and detected in the plane of sharp image of the scattering layer located at the focal plane of the probing beam, we have derived the equation

$$<P_s(x)> = \frac{cE_P}{2\pi a_0^2 f^2}\beta_\pi(x)\times$$

$$\int_{-\infty}^{+\infty} d^2\rho'' I_N(x,\boldsymbol{\rho}'')\int_D d^2\rho \tilde{I}_N(x,\boldsymbol{\rho}''+\boldsymbol{\rho}\,x/f)\,, \quad (1)$$

where E_P is the probing pulse energy, a_0 is the radius of the Gaussian beam in the plane $x=0$, f is the focal length of the receiving telescope, $\beta_\pi(x)$ is the backscattering coefficient,

$$I_N(x,\boldsymbol{\rho}'') = \left| \frac{1}{\lambda x} \int_{-\infty}^{+\infty} d^2\rho' \Pi_T(\boldsymbol{\rho}') \left\{ -\frac{\boldsymbol{\rho}'^2}{2a_0^2} + \right.\right.$$

$$\left.\left. + j[\psi_0(\boldsymbol{\rho}') - \tilde{\psi}_0(\boldsymbol{\rho}') - \frac{\pi}{\lambda F}\boldsymbol{\rho}'^2 + \frac{\pi}{\lambda x}(\boldsymbol{\rho}' - \boldsymbol{\rho}'')^2] \right\} \right|^2 \quad (2)$$

is the intensity of the probing beam focused at the distance F, $\Pi_T(\boldsymbol{\rho}')$ is the pupil function of the (ring) exit aperture of the transmitting telescope ($\Pi_T(\boldsymbol{\rho}') = 1$ at $r \le |\boldsymbol{\rho}'| \le R$ and $\Pi_T(\boldsymbol{\rho}') = 0$ at $r > |\boldsymbol{\rho}'| > R$; r is the inner radius and R is the outer radius of the aperture), $j = \sqrt{-1}$, $\psi_0(\boldsymbol{\rho}') = \sum_{i=0}^{\infty} C_i Z_i(\boldsymbol{\rho}')$ are aberrational phase (wave front) distortions, $-\tilde{\psi}_0(\boldsymbol{\rho}') = -\sum_{i=0}^{\infty} \tilde{C}_i Z_i(\boldsymbol{\rho}')$ is the phase change introduced by the deformable mirror, $Z_i(\boldsymbol{\rho}')$ is the Zernike polynomial, λ is the wavelength, $\int_D d^2\rho \equiv \int_{-d_R/2}^{d_R/2} dz \int_{-d_R/2}^{d_R/2} dy$,

$$\tilde{I}_N(x,\boldsymbol{\rho}'') = \left| \frac{1}{\lambda x} \int_{-\infty}^{+\infty} d^2\rho' \Pi_R(\boldsymbol{\rho}') \times \right.$$

$$\left. \exp\left\{ -j\frac{\pi}{\lambda F}\boldsymbol{\rho}'^2 + j\frac{\pi}{\lambda x}(\boldsymbol{\rho}' - \boldsymbol{\rho}'')^2 \right\} \right|^2,$$

and $\Pi_R(\boldsymbol{\rho}')$ is the pupil function of the entrance (circular) aperture of the receiving telescope ($\Pi_R(\boldsymbol{\rho}') = 1$ at $|\boldsymbol{\rho}'| \le r$ and $\Pi_R(\boldsymbol{\rho}') = 0$ at $|\boldsymbol{\rho}'| > r$).

To control the deformable mirror in order to compensate aberrational wave front distortions of the principal beam, it is necessary to measure the mean power of the scattered probing radiation. In the estimation of the mean power $\hat{P}_s$, we used the averaging over $N_x = \Delta x / \Delta p$ degrees of freedom along the optical axis, where Δx is the longitudinal dimension of the probed volume, and the accumulation of the echo signal over M sounding shots. The estimate can be represented as

$$\hat{P}_s = \langle \bar{P}_s \rangle (1 + \xi / \mathrm{SNR}), \quad (3)$$

where $\langle \bar{P}_s \rangle = \frac{1}{\Delta x} \int_{-\Delta x/2}^{+\Delta x/2} dx' \langle P_s(F + x') \rangle$ is the mean power of the radiation scattered from the probed volume near the focus ($x = F$), ξ is the Gaussian random variable with zero mean and unit variance,

$$\mathrm{SNR} = \frac{\langle \bar{P}_s \rangle \sqrt{N_x M}}{\sqrt{\langle \bar{P}_s \rangle^2 + \langle \bar{P}_s \rangle h\nu / (\eta \tau_P) + (\mathrm{NEP})^2 / (2\tau_P)}} \quad (4)$$

is the signal-to-noise ratio (ratio of the mean value of the useful component of the photocurrent proportional to $\langle \bar{P}_s \rangle$ to the rms deviation of the photocurrent), h is the Plank's constant, η is the quantum efficiency, and NEP is the noise equivalent power measured in W/Hz$^{1/2}$.

We simulated numerically the backscattered signal $\hat{P}_s$ based on Eqs. (1)–(4) on the assumption that $\psi_0(\boldsymbol{\rho}')$ is mostly determined by lowest-order aberrations and that in the series expansion of the phase in terms of the Zernike polynomials the consideration can be restricted to the first ten terms of the series. For the simulation of the process of aberration compensation by the deformable mirror through the variation of the coefficients $\tilde{C}_i$ ($1 \le i \le 9$) based on the backscattered signal, we used the iterative algorithm [1] for the determination of the maximum of the backscattered signal. The number of $\hat{P}_s$ estimates is three times greater than the number of iterations.

In the numerical simulation, we used λ = 1.06 µm, τ_P = 67 ns (Δp = 10 m), pulse repetition frequency PRF = 50 kHz, r = 10 cm, R = 50 cm, $C_i = (-1)^{i+1} 10$ ($i = 1, 2, ..., 9$), F = 10 km, field-of-view angle $\gamma = d_R / f$ = 10 µrad, η = 0.8, $\mathrm{NEP} = 10^{-15}$ W/Hz$^{1/2}$, N_x = 50 (Δx = 500 m), and M = 50. The initial height h_0 and the elevation angle φ were taken equal to 10 km and 90º, respectively (vertical path). According to the model proposed in [2], the backscattering coefficient at a height of 15 km is $\beta_\pi = 1.9 \cdot 10^{-8}$ m^{-1}sr^{-1}. The efficiency of the aberration compensation can be described by the parameter $\mu = I_0(x,0) / [I_0(x,0)|_{\tilde{C}_i = C_i}]$, where $I_0(x,0)|_{\tilde{C}_i = C_i}$ is the intensity of the principal beam in the far diffraction zone (x = 1000 km) at the optical path in the absence of aberrations.

Figure 2 shows the resultant signal-to-noise ratio, estimated power of the scattered radiation, and the normalized intensity of the principal beam as functions of the iteration number at probing pulse energy of 1 mJ and 4 mJ. One can see that at E_P = 4 mJ the parameter $\mu > 0.6$ after 110 iterations.

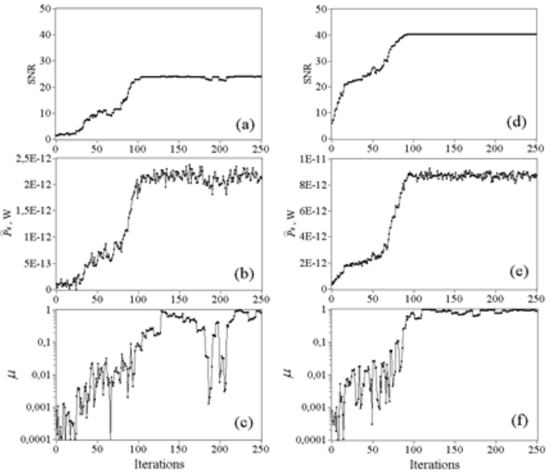

Fig. 2. Signal-to-noise ratio (a, d), estimated mean power of the scattered probing radiation (b, e), and normalized intensity of the principal beam (c, f) as functions of the iteration number at probing pulse energy of 1 mJ (a–c) and 4 mJ (d–f).

The analysis of 100 independent realizations has shown that the probability of compensation of aberrational beam distortions was 70% at E_P = 1 mJ and 100% at E_P = 4 mJ. The complete compensation of aberrations can be achieved at lower energy of the probing pulse, if a photodetector with the higher sensitivity is used. The condition $\mathrm{NEP}/E_P \le 2{,}5\cdot 10^{-13}$ c$^{1/2}$ should be valid.

2.2. CW laser

Figure 3 shows the schematic diagram of the setup for the compensation of wave front distortions of a CW laser beam based on the backscattered signal without splitting a laser beam into principal and probing ones.

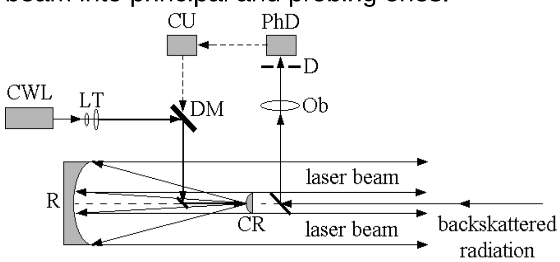

Fig. 3. Schematic of the setup: cw laser CWL, lens telescope LT, deformable mirror DM, contrreflector CR and reflector R of the radiating telescope, objective Ob of the receiving telescope, diaphragm D, photodetector PhD, control unit CU of the deformable mirror.

For this setup, the laser beam intensity I_0 at the point $\{x, \boldsymbol{\rho}''\}$, where x is the distance between the output aperture plane of the transmitting telescope and the observation plane $\boldsymbol{\rho}''=\{z'', y''\}$, is described by the equation $I_0(x,\boldsymbol{\rho}'') = [P_0/(\pi a_0^2)] I_N(x,\boldsymbol{\rho}'')$, where P_0 is power and a_0 is radius of the Gaussian beam at the plane $x=0$,

$$I_N(x,\boldsymbol{\rho}'') = \left| \frac{1}{\lambda x} \int_{-\infty}^{+\infty} d^2\rho' \Pi_T(\boldsymbol{\rho}') \exp\left\{ -\frac{\boldsymbol{\rho}'^2}{2a_0^2} + \right.\right.$$

$$\left.\left. + j[\psi_0(\boldsymbol{\rho}') - \tilde{\psi}_0(\boldsymbol{\rho}') + \frac{\pi}{\lambda x}(\boldsymbol{\rho}' - \boldsymbol{\rho}'')^2] \right\} \right|^2 \qquad (5)$$

is the normalized beam intensity, where beam wave front aberrations $\psi_0(\boldsymbol{\rho}')$ and the deformable mirror deformations $\tilde{\psi}_0(\boldsymbol{\rho}')$ are represented through Zernike polynomials, as in Eq. (2). The equation for the mean intensity of the scattered laser radiation I_s detected in the focal plane of the receiving telescope $\boldsymbol{\rho} = \{z, y\}$ has the form [1]

$$I_s(\boldsymbol{\rho}) = \frac{P_0}{\pi a_0^2 f^2} \int_0^{\infty} dx \beta_\pi(x) \times$$

$$\int_{-\infty}^{+\infty} d^2\rho'' I_N(x,\boldsymbol{\rho}'') \tilde{I}_N(x, \boldsymbol{\rho}'' + \boldsymbol{\rho} x/f), \qquad (6)$$

where f is the focal length of the receiving telescope, $\beta_\pi(x)$ is the backscattering coefficient,

$$\tilde{I}_N(x,\boldsymbol{\rho}'') = \left| \frac{1}{\lambda x} \int_{-\infty}^{+\infty} d^2\rho' \Pi_R(\boldsymbol{\rho}') \exp\left\{ j\frac{\pi}{\lambda x}(\boldsymbol{\rho}' - \boldsymbol{\rho}'')^2 \right\} \right|^2,$$

and $\Pi_R(\boldsymbol{\rho}')$ is the pupil function of the (circular) entrance aperture of the receiving telescope ($\Pi_R(\boldsymbol{\rho}') = 1$ at $|\boldsymbol{\rho}'| \le r$ and $\Pi_R(\boldsymbol{\rho}') = 0$ at $|\boldsymbol{\rho}'| > r$).

The scattered radiation is measured by a 4×4 photodetector array. Sensitive plates of the photodetectors are squares with the side length d_R. Then $\boldsymbol{\rho}_{kl} = \{z_{kl}, y_{kl}\}$ is the coordinate of the center of the sensitive plate of the kl th photodetector, where $k = 1, 2, 3, 4$ and $l = 1, 2, 3, 4$. The mean power of the scattered radiation detected by the kl th element of the array is determined as

$$P_{kl} = \int_{z_{kl}-d_R/2}^{z_{kl}+d_R/2} dz \int_{y_{kl}-d_R/2}^{y_{kl}+d_R/2} dy \ I_s(z,y). \qquad (7)$$

It follows from Eqs. (5) – (7) and Zernike polynomial expansions of the wave front aberrations and deformations, that P_{kl} and values of the normalized mean power of the scattered radiation $S_{kl} = 16 P_{kl} / \sum_{k=1}^{4}\sum_{l=1}^{4} P_{kl}$ depend on the difference of the vectors $\mathbf{C}$ and $\tilde{\mathbf{C}}$, where $\mathbf{C} = \{C_1, C_2, ..., C_i, ...\}$ and $\tilde{\mathbf{C}} = \{\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_i, ...\}$. With the photodetector noise taken into account, the normalized mean power of the scattered radiation $\hat{S}_{kl}$ can be estimated as

$$\hat{S}_{kl} = 16(P_{kl} + P_{NEP}\xi_{kl}) / \sum_{k=1}^{4}\sum_{l=1}^{4}(P_{kl} + P_{NEP}\xi_{kl}) , \quad (8)$$

where $P_{NEP} = \mathrm{NEP}\sqrt{B}$ is the noise equivalent power, B is the detector passband, ξ_{kl} is a random parameter obeying the Gaussian statistics with $< \xi_{kl} > = 0$ and $< \xi_{kl}\xi_{k'l'} > = \delta_{k-k'}\delta_{l-l'}$.

If, due to the deformable mirror operation, all 16 simultaneously recorded values $\hat{S}_{kl}$ becomes very close to the corresponding values $S_{kl}(0)$, then the laser beam wave front aberrations become compensated. The proposed algorithm for the control of the deformable mirror (to compensate aberrations) is based on the least square method. It consists essentially in the minimization of the functional

$$\rho(\tilde{\mathbf{C}}) = \sum_{k=1}^{4}\sum_{l=1}^{4}[\hat{S}_{kl}(\mathbf{C} - \tilde{\mathbf{C}}) - S_{kl}(\mathbf{0})]^2 \quad (9)$$

with the application of the iterative procedure. Consecutively, starting from $\tilde{C}_1$ and going from one variable $\tilde{C}_i$ to another, three values $\rho(\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_i, ..., \tilde{C}_I)$, $\rho(\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_i + \Delta C_i, ..., \tilde{C}_I)$, and $\rho(\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_i - \Delta C_i, ..., \tilde{C}_I)$ are measured (simulated) at any iteration. Here, ΔC_i is the step of variation of the i th Zernike coefficient. With the interpolation over three points, the minimum in ρ at the interval $[\tilde{C}_i - \Delta\tilde{C}_i, \tilde{C}_i + \Delta\tilde{C}_i]$ is determined. Accordingly to found minimum, the variable $\tilde{C}_i$ is changed and fixed. As index i achieves number l, we again pass to $\tilde{C}_1$, and repeat the minimization process. The number of ρ values is three times as great as the number of iterations. The values of $S_{kl}(0)$ are calculated by Eqs. (5)–(7) with the use of the information about the vertical profile of the backscattering coefficient, which can be obtained from the data of a lidar using the pulsed radiation at the same wavelength λ.

Effectiveness of this method was studied through the numerical simulation based on Eqs. (5) – (9) at the given parameters λ = 1.06 µm, r = 10 cm, R = 50 cm, $\gamma = d_R / f$ = 50 µrad, different aberrations (C_i, where $1 \le i \le 9$), and ratios P_{NEP} / P_0 on the assumption that the beam propagates vertically upward from the height h = 10 km. The vertical profile $\beta_\pi(h)$ was described by the model proposed in [2]. In the numerical simulation of the compensation process, we used the same criterion μ as in point 2.1.

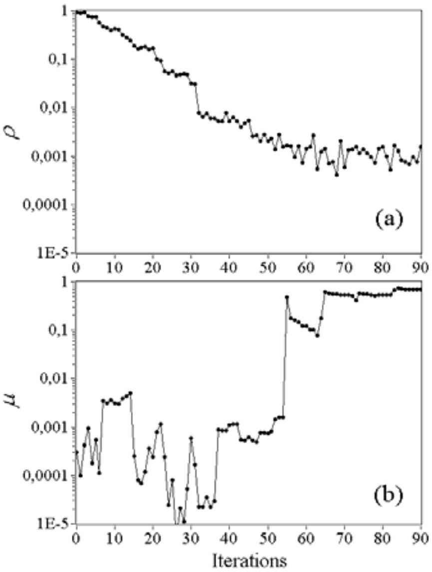

Fig.4. Dependence of the functional ρ (a) and the normalized beam intensity μ (b) on the iteration number at $C_1 = 12$, $C_2 = -13$ (tilt), $C_3 = 15$ (defocus), $C_4 = 10$, $C_5 = -9$ (astigmatism), $C_6 = 8$, $C_7 = 7$ (coma), $C_8 = 5$, $C_9 = -6$ (trefoil), and $P_{NEP} / P_0 = 10^{-16}$.

Figure 4 exemplifies the dependence of the functional ρ and the corresponding dependence of the normalized beam intensity μ on the iteration number. One can see that μ becomes greater than 0.5 starting from number 65. The analysis of the results of numerical simulations shows that aberrational distortions of the beam wave front can be compensated only at $P_{NEP} / P_0 \le 10^{-16}$.

The work was financially supported in part by the Russian Foundation for Basic Research, project 10-05-92505.

5. References

1. *Banakh V.A., Zhmylevski V.V., Ignat'ev A.B., Morozov V.V., Smalikho I.N.* Collimation of the Initial Wavefront of a Partially Coherent Light Beam by the Backscattered Signal // Opt. and Spectroscopy. 2010. Vol.108, No.1, pp.107–116.
2. *Banakh V.A., Zhmylevski V.V., Ignat'ev A.B., Morozov V.V., Smalikho I.N.* Pointing a partially coherent laser beam based on atmospheric backscattering / Atmos. Oceanic Opt. 2010. Vol. 23, No.10, pp.885-893.