Impact of Random Pointing Errors on Coherent Laser Radar Efficiency and Scintillation Index

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Introduction

We summarize work in progress [1] describing the impact of random pointing errors on coherent ladar performance. Pointing errors (random jitter and bias) reduce the coherent ladar mean signal power and increase signal scintillation. Performance depends on magnitude of the pointing errors compared to the size of the transmit beam, back-propagated local oscillator (BPLO) beam, the target and the degree to which the two beams are correlated in the target plane.

The target size plays a key role. When operating against an essentially infinitely wide target, the transmitter and receiver are never misaligned with the target regardless of the size of the pointing error. However, the transmitter and receiver may be misaligned with respect to each other. In the case of a very small target, misalignment between the beams and the target also plays a role.

Problem Overview

In this analysis we assume matched elliptical Gaussian transmit, '*t*' and BPLO, '*b*' with exp[-2] irradiance radii, ω_x and ω_y along the x and y axes. We also assume an elliptical diffuse Gaussian target, "*T*", centered at the origin, with peak power reflectance function $\rho_{\pi o}$ (sr^{-1}), exp[-2] radii ω_{Tx} and ω_{Ty}. By varying the target size relative to the beam size one is able to model point and extended targets with this Gaussian target model.

We also assume the beam motion is also Gaussian, with position bias (μ_{tx}, μ_{ty}) and (μ_{bx}, μ_{by}) for the transmit and BPLO beams, respectively. The rms pointing jitter along each axis, (σ_x, σ_y) and correlation between the transmit and BPLO positions (ρ_x, ρ_y) are assumed to be independent. Finally, this analysis assumes a short waveform integration time or duration compared to the LOS jitter time constant, such that the motion is essentially frozen during the receiver signal integration time.

In coherent ladar, the forward propagated (space and time) transmit and back propagated (space and time) LO field distributions interact with the target's complex scattering function to form the complex heterodyne signal [2]. Correlation between these two beam positions will depend on the ladar geometry and the round-trip time delay.

In a monostatic ladar it is reasonable to assume that the transmitter and receiver axis alignment is fixed. However, due to the time delay, this does not mean that the two beams will be correlated in the target-plane. When the round trip time to the target is small, compared to the pointing correlation time, the two beams will be correlated; however, at longer ranges they will be uncorrelated.

As an example, assume the power spectral density of the pointing is a Gaussian distribution with 3 dB bandwidth, *B*. Then the autocorrelation function will also be Gaussian with exp(-1) time equal to $\sqrt{\ln(2)}/\pi B$ ~ 0.265/*B*. Since the speed of light corresponds to a round trip time of 1 ms/150 km, we see that the correlation range is roughly 39,750 km-Hz/*B* or roughly 39.7 km for a 1 kHz 3 dB bandwidth pointing system, with a Gaussian PSD. Therefore, if the target range is much less than this range, the beams will be fully correlated and vice versa.

Theoretical Models

Static Efficiency Loss

Static pointing efficiency, η_o, is the ratio of the signal power achieved with static pointing error to the power with perfect alignment. For a general target and beams we can write

$$\eta_o(\vec{t},\vec{b}) = \frac{\int \rho_\pi(\vec{r}) I_t(\vec{r}-\vec{t}) I_b(\vec{r}-\vec{b}) d\vec{r}}{\int \rho_\pi(\vec{r}) I_t(\vec{r}) I_b(\vec{r}) d\vec{r}}, \quad (1)$$

where $\vec{t} = (t_x, t_y)$ and $\vec{b} = (b_x, b_y)$ are the vector positions of the transmit and BPLO beam centers and $\vec{r} = (x, y)$ represents a spatial coordinate in the target plane, with origin centered on the target. When evaluated with Gaussian beams and targets, the static efficiency evaluates to

$$\eta_o(\vec{t},\vec{b}) = \exp\frac{-2[\omega_{Tx}^2(t_x-b_x)^2+\omega_x^2(t_x^2+b_x^2)]}{\omega_x^2(\omega_x^2+2\omega_{Tx}^2)}$$
$$x\exp\frac{-2[\omega_{Ty}^2(t_y-b_y)^2+\omega_y^2(t_y^2+b_y^2)]}{\omega_y^2(\omega_y^2+2\omega_T^2)} .(2)$$

Generalized Moments

When the pointing errors are random, the n^{th} efficiency moment can be found by averaging the static efficiency, to the n^{th} power, over the distribution of pointing errors (i.e., via conditional statistics). That is,

$$<\eta^n> = \iint \eta_o^n(\vec{t},\vec{b})\, f(\vec{t};\vec{b})\, d\vec{t}\, d\vec{b} \ . \qquad (3)$$

For n = 1 and 2, this expression leads to the mean efficiency, η, and the normalized efficiency 2^{nd} moment, $\gamma^2 = E[\eta^2]/E[\eta]^2$.

When the beams, target, and pointing jitter are all elliptical Gaussian, with axes aligned, this moment integral is tractable and simplified analytic models can be realized, leading to physical insight even for non Gaussian problems.

Because Gaussian functions are x-y separable the moment integral (Eq. 3) is tractable provided the axes of the target, beam and motion all align. Even with this assumption, the integral solution is still quite cumbersome and a simplification in notation is realized through this x-y separability.

To this end we write the total efficiency as the product of two independent Cartesian terms, $\eta = \eta_x\eta_y$. Furthermore, the solution to the moment integral was found to be separable into a bias and jitter term. That is $\eta_x = \eta_{\mu x}\eta_{\sigma x}$ and $\eta_y = \eta_{\mu y}\eta_{\sigma y}$. Similar statements can be made about the 2^{nd} moment. That is $\gamma^2 = \gamma^2_x\gamma^2_y$, $\gamma^2_x = \gamma^2_{\mu x}\gamma^2_{\sigma x}$ and $\gamma^2_y = \gamma^2_{\mu y}\gamma^2_{\sigma y}$.

Finally, we recognize the symmetry between the two axial components of the moments and only present one and denote the solutions η_{xy} and γ^2_{xy}.

Gaussian Target Efficiency

For the generalized Gaussian target, the two part solution to the efficiency integral was determined to be given by [1]

$$\eta_{\mu_{x,y}} = \exp\left[-\frac{2[\omega^2(\mu_x^2+\mu_b^2)+4\sigma^2(\mu_x^2+\mu_b^2-2\rho\mu_x\mu_b)+\omega_T^2(\mu_x-\mu_b)^2]}{\omega^4+16\sigma^4(1-\rho^2)+8\omega^2\sigma^2+2\omega_T^2[\omega^2+4\sigma^2(1-\rho)]}\right] .(4)$$

and

$$\eta_{\sigma_{x,y}} = \sqrt{\frac{\omega^4+2\omega^2\omega_T^2}{\omega^4+16\sigma^4(1-\rho^2)+8\omega^2\sigma^2+2\omega_T^2[\omega^2+4\sigma^2(1-\rho)]}} .(5)$$

Again, this two part (bias and jitter) solution only applies to one axis of motion and bias and all the variables in these expressions correspond to those along one of the two axis (x or y). The complete solution is the product of two such terms.

When the beams, target, and motion are all circularly symmetric, and in the limit of small and large targets and fully correlated and uncorrelated beams, the total efficiency expression simplifies greatly.

For correlated beams, the total efficiency was determined to be given by the following two expressions for point and extended targets:

Point target, Correlated Beams

$$\eta|_{\rho=1} = \exp\left[-\frac{2[(|\vec{\mu}_t|^2+|\vec{\mu}_b|^2)}{\omega^2+8\sigma^2}\right] \times\exp\left[\frac{8(\sigma^2/\omega^2)(\vec{\mu}_t-\vec{\mu}_b)^2]}{\omega^2+8\sigma^2}\right]\frac{\omega^2}{\omega^2+8\sigma^2} . \qquad (6)$$

Extended target, Correlated Beams:

$$\eta|_{\rho=1} = \exp(-(\vec{\mu}_t-\vec{\mu}_b)^2/\omega^2) . \qquad (7)$$

The point target solution is a generalized version of Youmans' equation [3]. The difference being that he assumed identical biases ($\mu_t = \mu_b$). We note that the extended target solution is independent of beam jitter and only dependent on the beam bias vector difference length. For the point target there is a bias term and a term accounting for an increase in average "effective" beam size.

For uncorrelated beams, similar expressions are found.

Point target, Uncorrelated Beams

$$\eta|_{\rho=0} = \exp\left[-\frac{2(|\vec{\mu}_t|^2+|\vec{\mu}_b|^2)}{\omega^2+4\sigma^2}\right]\frac{\omega^4}{(\omega^2+4\sigma^2)^2} .(8)$$

Extended target, Uncorrelated Beams

$$\eta|_{\rho=0} = \exp\left[-\frac{(\vec{\mu}_t-\vec{\mu}_b)^2}{\omega^2+4\sigma^2}\right]\frac{\omega^2}{\omega^2+4\sigma^2} . \qquad (9)$$

With uncorrelated beams and zero bias, we see that, for the point target, the loss is roughly the square of the correlated beam loss and also exactly the square of the extended target loss. Furthermore, the extended target loss, for zero bias, is exactly the ratio of the beam area with and without random pointing errors.

Gaussian Target Scintillation Index with Speckle

The normalized signal power variance, δ, gives insight to signal power fluctuations about its mean value.

$$\delta^2 = \mathrm{var}[P]/\mu_P{}^2 = \gamma^2 - 1. \quad (10)$$

In addition to signal fluctuations imposed by random pointing, the heterodyne signal from a diffuse target also exhibits fluctuation due to the laser speckle phenomena. The mean loss due to speckle is mathematically captured in the signal overlap integral (Eq. 3). Because the normalized 2^{nd} moment of laser speckle is two, it is straight forward to show that laser speckle modulation doubles the random pointing contribution to the scintillation index. That is,

$$\delta_S^2 = 2\delta^2 + 1, \quad (11)$$

where $\delta_s{}^2$ is the scintillation index with speckle modulation.

Normalized 2^{nd} Moment

In a similar manner to solving for the mean efficiency, conditional statistics are employed for the normalized 2^{nd} movement. The solution, for the contribution of a single axis of motion and bias was found to be given by.

$$\gamma^2_{\mu_{x,y}} = \exp\left[-\frac{4[\omega_T^2(\mu_x-\mu_b)^2+\omega^2(\mu_x^2+\mu_b^2)+8\sigma^2(\mu_x^2+\mu_b^2-2\rho\mu_x\mu_b)]}{[\omega^2+8\sigma^2(1-\rho)][\omega^2+2\omega_T^2+8\sigma^2(1+\rho)]}\right] \times \exp\left[\frac{4[\omega_T^2(\mu_x-\mu_b)^2+\omega^2(\mu_x^2+\mu_b^2)+4\sigma^2(\mu_x^2+\mu_b^2-2\rho\mu_x\mu_b)]}{[\omega^2+4\sigma^2(1-\rho)][\omega^2+2\omega_T^2+4\sigma^2(1+\rho)]}\right] \quad (12)$$

and

$$\gamma^2_{\sigma_{x,y}} = \frac{[\omega^2+4\sigma^2(1-\rho)][\omega^2+2\omega_T^2+4\sigma^2(1+\rho)]}{\sqrt{[\omega^2+8\sigma^2(1-\rho)][\omega^2+2\omega_T^2+8\sigma^2(1+\rho)]}\sqrt{\omega^4+2\omega^2\omega_T^2}}. \quad (13)$$

As with the mean power loss, these normalized 2^{nd} moment results apply to elliptical beams, targets and motion provided the ellipse axes are all aligned.

Theoretical Predictions

In the figure below we provide plots of the theoretical efficiency and normalized variance or scintillation index, with speckle modulation. As can be seen the efficiency with large targets and correlated beams is one if the bias is zero, independent of the jitter. Performance degrades with bias. For small targets, the efficiency and scintillation is worse. *Good performance is seen to require that the bias and jitter be less than a fraction of a beam radius (approximately 0.25 for small targets to 0.5 for large targets)*.

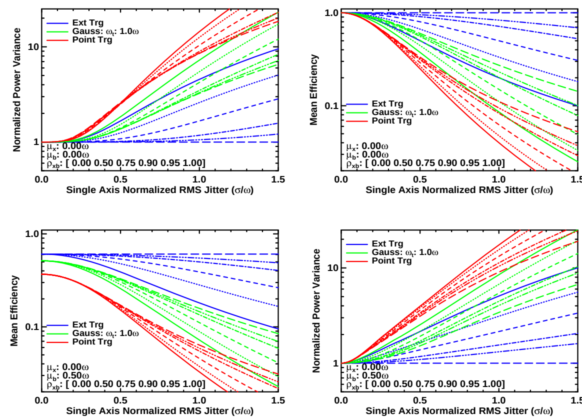

Figure 1. Theoretical efficiency and scintillation index with speckle, parametric in correlation vs. normalized single axis rms jitter (σ/ω), parametric in correlation (0 to 1) zero transmit bias for three BPLO biases: zero (left), 0.5ω (center) and 1.0 ω BPLO bias (right).

In Figure 2, we show performance as a function of beam correlation for a fixed rms jitter parametric in BPLO bias. As can be seen, point target performance only depends on the magnitude of the correlation and uncorrelated performance is worse than correlated performance. For extended targets, anti-correlation is the worst condition.

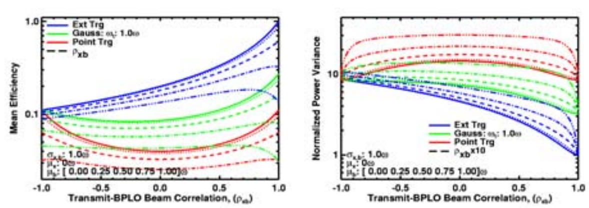

Figure 2. Efficiency (left) and scintillation index (right) vs. beam correlation for $\sigma = \omega$ and zero transmit beam bias parametric in BPLO bias ranging from zero to ω.

In the next figure we explore the behavior as a function of the round trip time to the target relative to the correlation time, T_c, corresponding to the exp(-1) time of the pointing autocorrelation function. In this analysis a Gaussian ACF was assumed. The corresponding pointing correlation coefficient, ρ, is shown as the dashed black line in Figure 3. This figure shows the efficiency and scintillation index as a function of the normalized round trip time, parametric in target type (extended, point, and Gaussian with $\omega_T = \omega$) and parametric in BPLO bias with zero transmit beam bias.

At short ranges ($t_r << T_c$) the beams are fully correlated and highest efficiency is generally achieved. Likewise the scintillation index is a minimum. For the extended target at short ranges, the scintillation index is dominated by speckle. Smaller targets yield higher scintillation and less efficiency. Bias tends to reduce efficiency and increase scintillation.

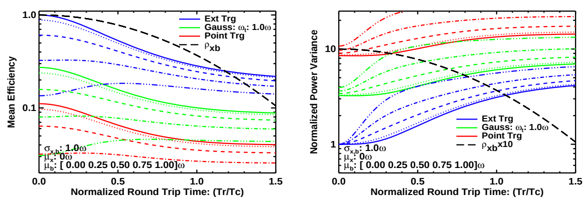

Figure 3. Efficiency and scintillation index as a function of the normalized round trip time, parametric in target type: extended (blue), point (red), and Gaussian (green) with $\omega_T = \omega$ and also parametric in BPLO bias with zero transmit beam bias. The Gaussian correlation coefficient is indicated by the parabolic dashed line.

Summary and Conclusions

In this work we presented analytic expressions for the pointing efficiency and scintillation index for a Gaussian beam coherent ladar with a diffuse Gaussian target reflectance profile and arbitrary correlation between the pointing errors of the transmitter and receiver. Simple cases representing a point target or an infinite extended target immediately follow by examining limiting cases of the Gaussian target.

Pointing jitter tends to decrease system efficiency and increase scintillation and the impact is worse for smaller targets. With correlated beams and extended targets, there is no performance loss with random pointing provided the bias is zero. In general, for high efficiency and low scintillation, both bias and jitter should be kept to a small fraction (< ¼ to ½) of a beam radius depending upon the target to beam size ratio (Figure 1)

Uncorrelated transmitter and receiver pointing errors, result in less efficiency and more scintillation than fully correlated errors and anti-correlated beam performance tends to be worse for large targets than uncorrelated errors. However for small targets, anti-correlated errors yield identical performance to correlated errors (Figure 2).

There is a square law efficiency relationship for uncorrelated errors between small and large targets (c.f., Eq 8 and 9). Likewise a similar square law relationship exists between the small target efficiency with correlated and uncorrelated errors (c.f., Eq. 6 and 8).

The efficiency and scintillation index expression given in this work are exact for Gaussian beam coherent ladar against Gaussian targets. The authors expect that, to some reasonable approximation, pointing efficiency behaves similarly in non-Gaussian coherent ladar against non-Gaussian targets. General rules-of-thumb found from this work are expected to apply well in other cases. Increasing pointing jitter seems likely to reduce system efficiency regardless of the beam profile or target shape, and the expression found in this work may give an idea of how quickly efficiency degrades. The formulas given here may assist in deriving tolerances when designing new coherent ladar systems. The lessons learned from examining Gaussian beam ladar are likely to help in designing better sensors and understanding existing system behavior.

References

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